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16. Abstract

Development of an approach to proving the validity of the dynamic method of studying the vibrations and stability of deformable systems. It is shown that the existence and uniqueness of the solution to the mixed problem describing the perturbed motions of such a system can be demonstrated on the basis of certain theorems for a linear eigenvalue problem which in the general case is a nonself-adjoint multiparametric problem.

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JUSTIFICATION OF THE DYNAMIC METHOD OF STUDYING ELASTIC SYSTEMS

Ya. S. Podstrigach[1], D. A. Baydak and L. M. Zoriy

Questions of the basis for the dynamic method, which plays a basic role in the study of the vibrations and stability of deformable systems, have fundamental value for the general theory and its applications. However, for systems with distributed parameters, such questions are in the initial stage of development [1-4]. /1049*

The approach developed here to justify the dynamic method is applicable to elastic systems, investigation of whose perturbations leads to a mixed problem of the form

$$\begin{gathered}\mathcal{L}[u(x,t)]+g(x)[\partial^2u/\partial t^2+2c\ \ \partial u/\partial t+au]=F(u,\partial u/\partial x,\ldots,\ \partial^{2n}u/\partial x^{2n}),\\ U_{i0}[u(x,t)]_{x=0}=0,\quad U_{i1}[u(x,t)]_{x=1}=0,\quad i=1,2,\ldots,n;\\ u(x,0)=\varphi(x),\quad \partial u(x,0)/\partial t=\psi(x),\quad 0\leqslant x\leqslant 1,\end{gathered} \tag{1}$$

where $\mathcal{L}$ is a linear differential expression of the $2n^{th}$ order with respect to the variable x; u(x, t) is a displacement of the system being considered; the function $g(x) > 0$; the constants a and c are non-negative; F is a certain non-linear function such that $F \equiv 0$ when $u \equiv 0$; U_{i0} and U_{i1} are linear forms with respect to $u, u_x, \ldots, u_{x^{2n-1}}$; and $\varphi(x)$ and $\psi(x)$ are any assigned functions. The coefficients of the differential expression and of the linear forms in problem (1) are dependent on the variable x and the parameters of the loads applied to the system.

The perturbations u(x, t) are assumed to belong to a set, U, of eigen functions which are continuous with respect to a set of arguments/of functions continuously differentiable up to the order 2n inclusively with respect to x and up to the second order with respect to $t \in [0, T]$, and which satisfy the boundary conditions of problem (1), with the norm

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*Numbers in the margin indicate pagination in the foreign text.

$$\|u\| = \left(\int_0^1 g(x)\,|u(x,t)|^2\,dx \right)^{1/2}.$$

Definition. A null solution of problem (1) is called stable if, for all values of ε_1, $\varepsilon_2 > 0$, there exist corresponding values of δ_1, $\delta_2 > 0$, such that $||u|| < \varepsilon_1$, and $||\partial u/\partial t|| < \varepsilon_2$ when $t > 0$, if $||\varphi|| < \delta_1$ and $||\psi|| < \delta_2$. If then this solution is stable, and, in addition, $||u|| \to 0$, $||\partial u/\partial t|| \to 0$ when $t \to \infty$, then it is called asymptotic stability.

Later on, the linear problem corresponding to (1) in eigen values determines:

$$\mathscr{L}[y(x)] - \omega^2 g(x) y(x) = 0,$$

$$U_{i0}[y(x)]_{x=0} = 0, \quad U_{i1}[y(x)]_{x=1} = 0, \quad i = 1, 2, \ldots, n. \tag{2}$$

In the general case it proves to be a non-self-adjoint multiparametric problem. /1050

Further, for the sake of simplicity, it is considered that its real values depend on a single load parameter, p. Obviously, when $p = 0$ they are positive; we shall consider them simple. Then there exists a half-interval of variation, $I = [0, p^*)$, in the parameter p such that[2]

$$0 < \omega_1^2(p) < \omega_2^2(p) < \ldots < \omega_n^2(p) < \ldots, \qquad p \in I.$$

The sequences of eigen functions $\{y_m(x)\}_{x=1}^{\infty}$ and $\{z_n(x)\}_{x=1}^{\infty}$ of problem 2 and the conjugate to it, as is known, form a bi-orthogonal system of functions with a weight $g(x)$, $0 \leq x \leq 1$; and each of these sequences individually is a Reiss base in the space $L_g^2(0, 1)$ [5].

We assume that the nonlinearity F and the initial functions $\varphi(x)$ and $\psi(x)$ satisfy the conditions

$$\|F_1 - F_2\| \leqslant G(t)\,\|u_1 - u_2\|^{1+\alpha} \quad \forall u_1, u_2 \in U,$$

where F_1 and F_2 are its values when $u = u_1$ and $u = u_2$, respectively; and $G(t)$ is any continuous positive function, whose characteristic index $\gamma^* \leq c$; $\alpha \leq 0$ is a certain constant.

The functions $\varphi(x)$ and $\psi(x)$ are expanded according to the real functions of problem (2) into regularly converging series; the coefficients of these expansions belong to a compact set in the space ℓ_2; the series

$$\sum_{n=1}^{\infty} \omega_n^2 \varphi_n y_n(x), \; \varphi_n = (\varphi, y_n); \quad \sum_{n=1}^{\infty} \omega_n^2 \psi_n y_n(x), \; \psi_n = (\psi, y_n).$$

converge

[2]The value p^* of the loading parameter corresponds to an Eulerian, or self-vibrational, form of stability loss.

Then the following theorems are valid.

Theorem 1. When $p \in I$ or $p = p^* + \nu$ ($\nu > 0$, a rather small number), and $0 \leq t \leq T$, there is a unique solution of problem (1), $u(x, t) \in U$, which can be represented in the form of the series:

$$u(x,t)=\sum_{n=1}^{\infty} x_n(t) y_n(x),$$

wherein the estimate of a lower and an upper bound occurs

$$m\|u\|^2 \leqslant \sum_{n=1}^{\infty} |x_n|^2 \leqslant M\|u\|^2,$$

where m and M are constants which do not depend on u.

Theorem 2. If $p < p^*$ and $c < 0$, then the null solution of problem (1) is asymptotically stable.

Theorem 3. If $p = p^* + \nu$, then the null solution of problem (1) is unstable.

Proof of the existence and uniqueness of the solution to the problem considered is conducted by reducing it to an equivalent problem for a denumerable system of integral equations obtainable on the basis of the above-noted properties of the eigen functions, with subsequent use of the contractive mappings principle.

Theorems 2 (and 3) about (in)stability are proved to a first approximation by applying the direct Lyapunov method to the appropriate denumerable system of differential equations.

We note that this approach to justifying the dynamic method is being /1051
extended to more complex problems for deformable systems with distributed parameters.

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